Precalculus Name_____ ID: 1

Solving Trig Equations Review Problems

Date_____ Period____

Solve each equation for $0 \le \theta < 2\pi$.

1) $2\sec^2\theta = 4 + \sec^2\theta$

2) $1 = 2\cos^2\theta$

3) $-2\cos\theta = -\sqrt{3}\cos\theta\csc\theta$

4) $3\tan\theta\cot\theta - \tan\theta = \sqrt{3}\tan\theta - \tan\theta$

5) $-1 = 2\tan\theta + \tan^2\theta$

6) $-2\sin^2\theta + 2 = 3\sin\theta + 3$

7) $0 = -\sin\theta + \sin^2\theta - \cos^2\theta$

8) $0 = 3\csc\theta + 3 + \cot^2\theta$

9) $\csc^2\theta - 1 = 0$

10) $\cos\theta - 1 = \sin\theta - 2$

11) $8 = \cos 2\theta + 10\cos^2\theta$

12) $\sin 2\theta = -\cos\theta + 2\sin 2\theta$

13) $-6\sin^2\theta = \cos 2\theta - 4$

14) $10 = \cos 2\theta + 14\sin^2\theta$

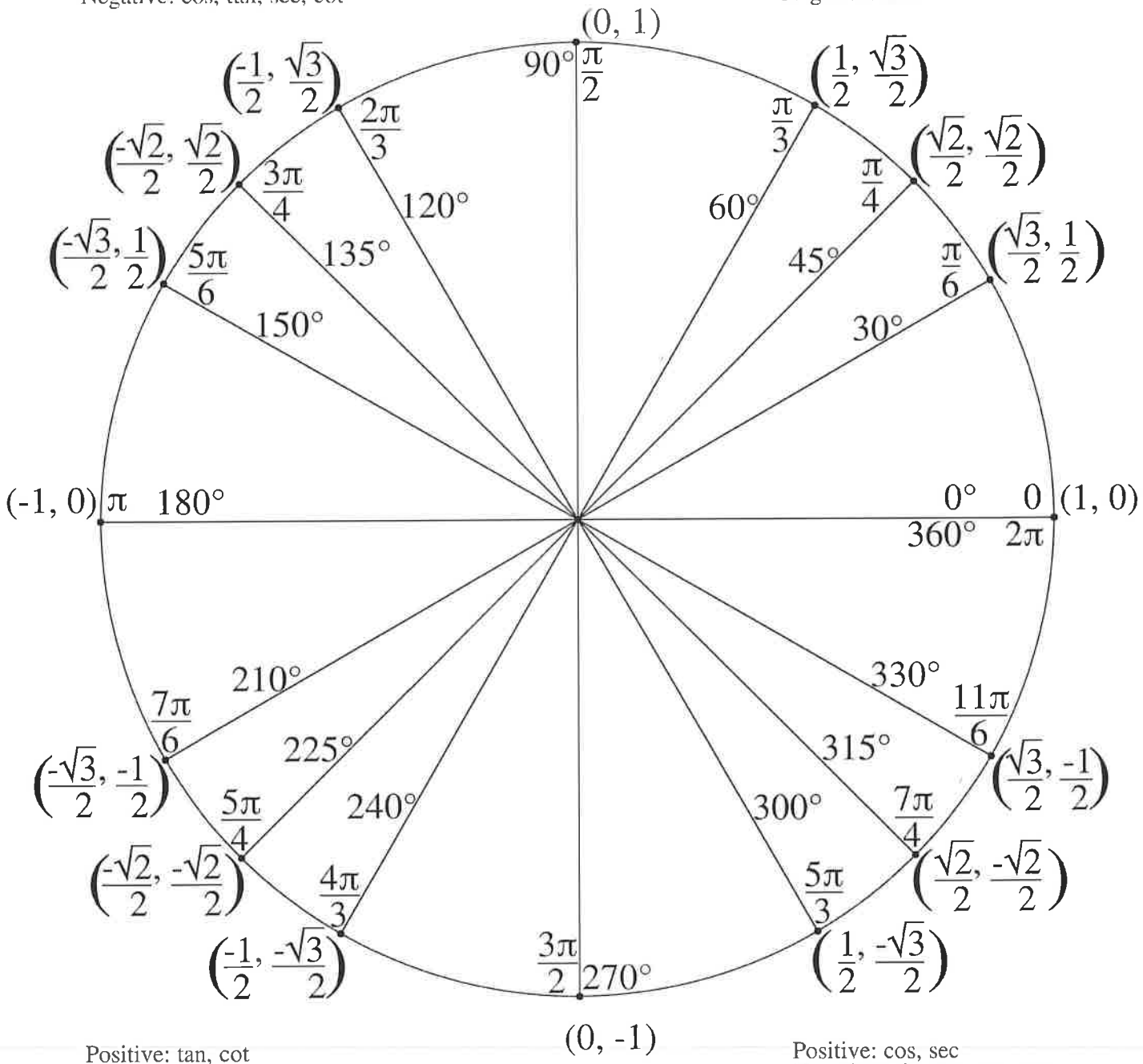
15) $-2 + 2\cos^2\theta = -\cos 2\theta$

16) $3\sin 2\theta = 2\cos\theta + 2\sin 2\theta$

The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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Solving Trig Equations Review Problems

① $2\sec^2\theta = 4 + \sec^2\theta$

$\sqrt{\sec^2\theta} = \sqrt{4}$

$\sec\theta = \pm 2$

$\cos\theta = \pm \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

② $\frac{1}{2} = \frac{2\cos^2\theta}{2}$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm \frac{1}{\sqrt{2}}$

$\cos\theta = \pm \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

③ $-2\cos\theta = -\sqrt{3}\cos\theta\csc\theta$

$-2\cos\theta + \sqrt{3}\cos\theta\csc\theta = 0$

$\cos\theta(-2 + \sqrt{3}\csc\theta) = 0$

$\cos\theta = 0$ $\quad -2 + \sqrt{3}\csc\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sqrt{3}\csc\theta = 2$

$\csc\theta = \frac{2}{\sqrt{3}}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

④ $3\tan\theta\cot\theta - \tan\theta = \sqrt{3}\tan\theta - \tan\theta$

$3(1) - \tan\theta = \sqrt{3}\tan\theta - \tan\theta$

$\frac{3}{\sqrt{3}} = \frac{\sqrt{3}\tan\theta}{\sqrt{3}}$

$\tan\theta = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\theta = \frac{3\sqrt{3}}{3}$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

⑤ $-1 = 2\tan\theta + \tan^2\theta$

$0 = \tan^2\theta + 2\tan\theta + 1$

$0 = (\tan\theta + 1)(\tan\theta + 1)$

$\tan\theta + 1 = 0$

$\tan\theta = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

⑥ $-2\sin^2\theta + 2 = 3\sin\theta + 3$

$-1(-2\sin^2\theta - 3\sin\theta - 1 = 0)$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$2\sin\theta + 1 = 0 \quad \sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2} \quad \sin\theta = -1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \theta = \frac{3\pi}{2}$

⑦ $0 = -\sin\theta + \sin^2\theta - \cos^2\theta$

$0 = -\sin\theta + \sin^2\theta - (1 - \sin^2\theta)$

$0 = -\sin\theta + \sin^2\theta - 1 + \sin^2\theta$

$0 = 2\sin^2\theta - \sin\theta - 1$

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$2\sin\theta + 1 = 0 \quad \sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{2} \quad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \theta = \frac{\pi}{2}$

⑧ $0 = 3\csc\theta + 3 + \cot^2\theta$

$0 = 3\csc\theta + 3 + \csc^2\theta - 1$

$0 = \csc^2\theta + 3\csc\theta + 2$

$0 = (\csc\theta + 2)(\csc\theta + 1)$

$\csc\theta + 2 = 0 \quad \csc\theta + 1 = 0$

$\csc\theta = -2 \quad \csc\theta = -1$

$\sin\theta = -\frac{1}{2} \quad \sin\theta = -1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \theta = \frac{3\pi}{2}$

(9) $\csc^2\theta - 1 = 0$

$\sqrt{\csc^2\theta} = \sqrt{1}$

$\csc\theta = \pm 1$

$\sin\theta = \pm 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

(10) $\cos\theta - 1 = \sin\theta - 2$

$+1 \qquad +1$

$(\cos\theta)^2 = (\sin\theta - 1)^2$

$\cos^2\theta = \sin^2\theta - 2\sin\theta + 1$

$+ -\sin^2\theta = \sin^2\theta - 2\sin\theta + 1$

$0 = 2\sin^2\theta - 2\sin\theta$

$0 = 2\sin\theta(\sin\theta - 1)$

$2\sin\theta = 0 \qquad \sin\theta - 1 = 0$

$\sin\theta = 0 \qquad \sin\theta = 1$

$\theta = 0, \pi \qquad \theta = \frac{\pi}{2}$

extraneous

Here's why!

@ 0 radians
(1, 0)

$\cos\theta - 1 = \sin\theta - 1$

$\cos 0 - 1 = \sin 0 - 1$

$1 - 1 = 0 - 1$

$0 \neq -1$

(11) $8 = \cos 2\theta + 10\cos^2\theta$

$8 = 2\cos^2\theta - 1 + 10\cos^2\theta$

$8 = 12\cos^2\theta - 1$

$\frac{9}{12} = \frac{12\cos^2\theta}{12}$

$\frac{9}{12} = \cos^2\theta$

$\pm\frac{3}{2\sqrt{3}} = \cos^2\theta$

$\pm\frac{3\sqrt{3}}{6} = \cos\theta$

$\pm\frac{\sqrt{3}}{2} = \cos\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(12) $\sin 2\theta = -\cos\theta + 2\sin 2\theta$

$-\sin 2\theta \qquad -\sin 2\theta$

$0 = -\cos\theta + \sin 2\theta$

$0 = -\cos\theta + 2\sin\theta\cos\theta$

$0 = -\cos\theta(1 - 2\sin\theta)$

$-\cos\theta = 0 \qquad 1 - 2\sin\theta = 0$

$\cos\theta = 0 \qquad -2\sin\theta = -1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(13) $-6\sin^2\theta = \cos 2\theta - 4$

$-6\sin^2\theta = 1 - 2\sin^2\theta - 4$

$-4\sin^2\theta = -3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(14) $10 = \cos 2\theta + 14\sin^2\theta$

$10 = 1 - 2\sin^2\theta + 14\sin^2\theta$

$10 = 1 + 12\sin^2\theta$

$9 = 12\sin^2\theta$

$\sqrt{\frac{9}{12}} = \sqrt{\sin^2\theta}$

$\sin\theta = \pm\frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\sin\theta = \pm\frac{3\sqrt{3}}{6}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(15) $-2 + 2\cos^2\theta = -\cos 2\theta$

$-2 + 2\cos^2\theta = -(2\cos^2\theta - 1)$

$-2 + 2\cos^2\theta = -2\cos^2\theta + 1$

$4\cos^2\theta = 3$

$\sqrt{\cos^2\theta} = \sqrt{\frac{3}{4}}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(16) $3\sin 2\theta = 2\cos\theta + 2\sin 2\theta$

$-2\sin 2\theta \qquad -2\sin 2\theta$

$\sin 2\theta = 2\cos\theta$

$2\sin\theta\cos\theta - 2\cos\theta = 0$

$2\cos\theta(\sin\theta - 1) = 0$

$2\cos\theta = 0 \qquad \sin\theta - 1 = 0$

$\cos\theta = 0 \qquad \sin\theta = 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \theta = \frac{\pi}{2}$