

Solve each equation for the given interval

1) $\sin 2\theta = \cos \theta, 90^\circ \leq \theta < 180^\circ$

$$\begin{aligned} \sin 2\theta - \cos \theta &= 0 \\ 2\sin \theta \cos \theta - \cos \theta &= 0 \\ \cos \theta (2\sin \theta - 1) &= 0 \end{aligned}$$

$\cos \theta = 0$

$\theta = 90^\circ$

$2\sin \theta - 1 = 0$

$2\sin \theta = 1$
 $\sin \theta = \frac{1}{2}$

$\theta = 150^\circ$

2) $\sqrt{2} \cos \theta = \sin 2\theta, 0^\circ \leq \theta, 360^\circ$

$$\begin{aligned} \sqrt{2} \cos \theta - \sin 2\theta &= 0 \\ \sqrt{2} \cos \theta - 2\sin \theta \cos \theta &= 0 \\ \cos \theta (\sqrt{2} - 2\sin \theta) &= 0 \end{aligned}$$

$\cos \theta = 0$

$\theta = 90^\circ$

$\theta = 270^\circ$

$\sqrt{2} - 2\sin \theta = 0$

$-2\sin \theta = -\sqrt{2}$

$\sin \theta = \frac{\sqrt{2}}{2}$

$\theta = 45^\circ$

$\theta = 135^\circ$

3) $\cos \theta + \cos(90 - \theta) = 0, 0 \leq \theta < 2\pi$

$$\begin{aligned} \cos \theta + \cos 90 \cos \theta + \sin 90 \sin \theta &= 0 \\ \cos \theta + \sin \theta &= 0 \\ \cos \theta &= -\sin \theta \end{aligned}$$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

4) $\tan^2 \theta + \sec \theta = 1, \frac{\pi}{2} \leq \theta < \pi$

$$\begin{aligned} \sec^2 \theta - 1 + \sec \theta &= 1 \\ \sec^2 \theta + \sec \theta - 2 &= 0 \\ (\sec \theta + 2)(\sec \theta - 1) &= 0 \end{aligned}$$

$\sec \theta = -2$

$\cos \theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

$\sec \theta = 1$

$\cos \theta = 1$

No Solution

5) $2 + \cos \theta = 2\sin^2 \theta, \pi \leq \theta \leq \frac{3\pi}{2}$

$$\begin{aligned} 2 + \cos \theta - 2(1 - \cos^2 \theta) &= 0 \\ +\cos \theta - 2 + 2\cos^2 \theta &= 0 \\ 2\cos^2 \theta + \cos \theta - 2 &= 0 \\ \cos \theta (2\cos \theta + 1) &= 0 \end{aligned}$$

$\cos \theta = 0$

$\theta = \frac{3\pi}{2}$

$2\cos \theta + 1 = 0$

$2\cos \theta = -1$

$\cos \theta = -\frac{1}{2}$

$\theta = \frac{4\pi}{3}$

Solve each equation for all values of θ if θ is measured in radians.

6) $\cos^2 \theta = \sin^2 \theta$

$$\cos^2 \theta - \sin^2 \theta = 0$$

$$\cos 2\theta = 0$$

$$2\cos^2 \theta - 1 = 0$$

$$2\cos^2 \theta - 1 = 0$$

$$2\cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \frac{1}{\sqrt{2}}$$

$$\cos \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4} + \pi n$$

$$\frac{3\pi}{4} + \pi n$$

7) $\cos^2 \theta \sin \theta = \sin \theta$

$$\cos^2 \theta \sin \theta - \sin \theta = 0$$

$$\sin \theta (\cos^2 \theta - 1) = 0$$

$$\sin \theta = 0$$

$$\theta = 0 + \pi n$$

$$\cos^2 \theta - 1 = 0$$

$$\cos^2 \theta = 1$$

$$\cos \theta = \pm 1$$

$$\theta = 0 + \pi n$$

8) $2 \cos 2\theta = 1 - 2 \sin^2 \theta$

$$2(1 - 2\sin^2 \theta) = 1 - 2\sin^2 \theta$$

$$2 - 4\sin^2 \theta = 1 - 2\sin^2 \theta$$

$$1 - 2\sin^2 \theta = 0$$

$$-2\sin^2 \theta = -1$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4} + \pi n$$

$$\frac{3\pi}{4} + \pi n$$

9) $\csc^2 \theta - 3 \csc \theta + 2 = 0$

$$(\csc \theta - 2)(\csc \theta - 1) = 0$$

$$\csc \theta = 2 \quad \csc \theta = 1$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \frac{\pi}{6} + 2\pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n$$

$$\theta = \frac{\pi}{2} + 2\pi n$$

10) $4 \sin^2 \theta = 3$

$$\sin^2 \theta = \frac{3}{4}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3} + \pi n$$

$$\theta = \frac{2\pi}{3} + \pi n$$

11) $4 \sin^2 \theta - 1 = 0$

$$4 \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + \pi n$$

$$\theta = \frac{5\pi}{6} + \pi n$$

12) $\cos 2\theta + \sin \theta - 1 = 0$

$$1 - 2\sin^2 \theta + \sin \theta - 1 = 0$$

$$-2\sin^2 \theta + \sin \theta = 0$$

$$\sin \theta (-2\sin \theta + 1) = 0$$

$$\sin \theta = 0$$

$$\theta = 0 + \pi n$$

$$-2\sin \theta = -1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + 2\pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n$$