Precalculus   Name _Cornely_   ID: 1

Solving Trig Equations Day 2

Date_____ Period____

Solve each equation for all values of θ where θ is measured in radians.

1) $\cos^2 \theta + 3 = 4$

$\cos^2 \theta = 1$

$\cos \theta = \pm 1$

$\boxed{\theta = 0 + \pi n}$

2) $0 = -1 + \tan^2 \theta$ $\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{2} n}$

$\tan^2 \theta = 1$

$\tan \theta = \pm 1$

3) $3 = 4\sin^2 \theta$

$\sin^2 \theta = \frac{3}{4}$

$\sin \theta = \pm \frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{3} + \pi n}$

$\boxed{\theta = \frac{2\pi}{3} + \pi n}$

4) $3\sin^2 \theta = 1 + \sin^2 \theta$ $\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{2} n}$

$2\sin^2 \theta = 1$

$\sin^2 \theta = \frac{1}{2}$

$\sin \theta = \pm \frac{\sqrt{2}}{2}$

5) $2\cos \theta = -\sqrt{3}\cos \theta + \cos \theta \tan \theta + 2\cos \theta$

$\sqrt{3}\cos \theta - \cos \theta \tan \theta = 0$

$\cos \theta (\sqrt{3} - \tan \theta) = 0$

$\cos \theta = 0 \qquad \tan \theta = \sqrt{3}$

$\boxed{\theta = \frac{\pi}{2} + \pi n} \quad \boxed{\theta = \frac{\pi}{3} + \pi n}$

6) $3\sin \theta \tan \theta - 2\tan \theta = \sqrt{3}\sin \theta - 2\tan \theta$

$3\sin \theta \tan \theta - \sqrt{3}\sin \theta$

$\sin \theta (3\tan \theta - \sqrt{3}) = 0$

$\sin \theta = 0 \qquad 3\tan \theta = \sqrt{3}$

$\boxed{\theta = 0 + \pi n} \qquad \tan \theta = \frac{\sqrt{3}}{3}$

$\boxed{\theta = \frac{\pi}{6} + \pi n}$

7) $\sin \theta + 2\sin^2 \theta = 0$

$\sin \theta (1 + 2\sin \theta) = 0$

$\sin \theta = 0 \qquad 1 + 2\sin \theta = 0$

$\boxed{\theta = 0 + \pi n} \qquad 2\sin \theta = -1$

$\sin \theta = -\frac{1}{2}$

$\boxed{\theta = \frac{7\pi}{6} + 2\pi n}$

$\boxed{\theta = \frac{11\pi}{6} + 2\pi n}$

8) $\cos \theta \tan \theta - 2\cos \theta = -\cos \theta$

$\qquad\qquad\qquad +\cos \theta \quad +\cos \theta$

$\cos \theta \tan \theta - \cos \theta = 0$

$\cos \theta (\tan \theta - 1) = 0$

$\cos \theta = 0 \qquad \tan \theta - 1 = 0$

$\boxed{\theta = \frac{\pi}{2} + \pi n} \qquad \tan \theta = 1$

$\boxed{\theta = \frac{\pi}{4} + \pi n}$

9) $4\cos^2 \theta - 1 = -\cos \theta + 2\cos^2 \theta$

$-2\cos^2 \theta \qquad\qquad -2\cos^2 \theta$

$2\cos^2 \theta - 1 = -\cos \theta$

$2\cos^2 \theta + \cos \theta - 1 = 0$

$(2\cos \theta - 1)(\cos \theta + 1) = 1$

$2\cos \theta - 1 = 0 \qquad \cos \theta + 1 = 0$

$\cos \theta = \frac{1}{2} \qquad\qquad \cos \theta = -1$

$\boxed{\theta = \frac{\pi}{3} + 2\pi n}$

$\boxed{\theta = \frac{5\pi}{3} + 2\pi n}$

$\boxed{\theta = \pi + 2\pi n}$

10) $2\tan \theta - \tan^2 \theta = 1$

$\tan^2 \theta - 2\tan \theta + 1 = 0$

$(\tan \theta - 1)(\tan \theta - 1) = 0$

$\tan \theta - 1 = 0$

$\tan \theta = 1$

$\boxed{\theta = \frac{\pi}{4} + \pi n}$

-1-

11) $2\sin\theta + 2 = \cos^2\theta$

$2\sin\theta + 2 = 1 - \sin^2\theta$

$\sin^2\theta + 2\sin\theta + 1 = 0$

$(\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$$\boxed{\theta = \frac{3\pi}{2} + 2\pi n}$$

12) $-3\sin\theta + 2 = \cos^2\theta - \sin^2\theta$

$-3\sin\theta + 2 = 1 - \sin^2\theta - \sin^2\theta$

$-3\sin\theta + 2 = 1 - 2\sin^2\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0 \qquad \sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2} \qquad\qquad \sin\theta = 1$

$$\boxed{\theta = \frac{\pi}{6} + 2\pi n}$$
$$\boxed{\theta = \frac{5\pi}{6} + 2\pi n}$$

$$\boxed{\theta = \frac{\pi}{2} + 2\pi n}$$

13) $\cos^2\theta - \sin^2\theta = -\cos\theta$

$\cos^2\theta - (1 - \cos^2\theta) = -\cos\theta$

$\cos^2\theta - 1 + \cos^2\theta = -\cos\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0 \qquad \cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2} \qquad\qquad \cos\theta = -1$

$$\boxed{\begin{array}{l}\theta = \frac{\pi}{3} + 2\pi n \\[4pt] \theta = \frac{5\pi}{3} + 2\pi n \\[4pt] \theta = \pi + 2\pi n\end{array}}$$

14) $2 + 2\cos\theta = \sin^2\theta$

$2 + 2\cos\theta = 1 - \cos^2\theta$

$\cos^2\theta + 2\cos\theta + 1 = 0$

$(\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta + 1 = 0$

$\cos\theta = -1$

$$\boxed{\theta = \pi + 2\pi n}$$

15) $3\cos\theta = \sin\theta - 1 + 4\cos\theta$

$\quad -4\cos\theta \qquad\qquad -4\cos\theta$

$(-\cos\theta)^2 = (\sin\theta - 1)^2 \quad\leftarrow \text{FOIL}$

$\cos^2\theta = \sin^2\theta - 2\sin\theta + 1$

$1 - \sin^2\theta = \sin^2\theta - 2\sin\theta + 1$

$2\sin^2\theta - 2\sin\theta = 0$

$2\sin\theta(\sin\theta - 1) = 0$

$2\sin\theta = 0 \qquad \sin\theta = 1$

$\sin\theta = 0$

$$\boxed{\begin{array}{l}\theta = 0 + \pi n \\[4pt] \theta = \frac{\pi}{2} + 2\pi n\end{array}}$$

16) $\sqrt{3}\cos\theta - \sin\theta = -2\sin\theta$

$\quad +\sin\theta \quad +\sin\theta$

$(\sqrt{3}\cos\theta)^2 = (-\sin\theta)^2$

$3\cos^2\theta = \sin^2\theta$

$3\cos^2\theta = 1 - \cos^2\theta$

$4\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$

$$\boxed{\begin{array}{l}\theta = \frac{\pi}{3} + \pi n \\[4pt] \theta = \frac{2\pi}{3} + \pi n\end{array}}$$

17) $1 - 2\cos\theta = -3\cos\theta - \sin\theta$

$\quad +3\cos\theta \quad +3\cos\theta$

$(1 - \cos\theta)^2 = (-\sin\theta)^2$

$1 - 2\cos\theta + \cos^2\theta = \sin^2\theta$

$1 - 2\cos\theta + \cos^2\theta = 1 - \cos^2\theta$

$2\cos^2\theta - 2\cos\theta = 0$

$2\cos\theta(\cos\theta - 1) = 0$

$2\cos\theta = 0 \qquad \cos\theta - 1 = 0$

$\cos\theta = 0 \qquad\qquad \cos\theta = 1$

$$\boxed{\begin{array}{l}\theta = \frac{\pi}{2} + \pi n \\[4pt] \theta = 0 + 2\pi n\end{array}}$$

18) $\sqrt{3}\sin\theta + \sin\theta = -\cos\theta + \sin\theta$

$3\sin^2\theta = \cos^2\theta$

$3\sin^2\theta = 1 - \sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$$\boxed{\begin{array}{l}\theta = \frac{\pi}{6} + \pi n \\[4pt] \theta = \frac{5\pi}{6} + \pi n\end{array}}$$