Precalculus | Name Cornely | ID: 1

Fundamental and Double Angles QUIZ Review | Date______ | Period____

Use identities to find the value of each expression.

1) Find $\csc\theta$ and $\cot\theta$ if $\tan\theta = -\frac{1}{3}$ and $\sec\theta > 0$.

$\boxed{\cot\theta = -3 \quad \csc\theta = -\sqrt{10}}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + (-3)^2 = \csc^2\theta$

$1 + 9 = \csc^2\theta$

$10 = \csc^2\theta$

$\csc\theta = \pm\sqrt{10}$

2) Find $\cot\theta$ and $\csc\theta$ if $\tan\theta = \frac{1}{2}$ and $\sin\theta < 0$.

$\boxed{\cot\theta = 2 \quad \csc\theta = -\sqrt{5}}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + 2^2 = \csc^2\theta$

$5 = \csc^2\theta$

$\csc\theta = \pm\sqrt{5}$

3) Find $\sec\theta$ and $\sin\theta$ if $\tan\theta = -\frac{2}{3}$ and $\sec\theta < 0$.

$\boxed{\sec\theta = \frac{-\sqrt{13}}{3} \quad \sin\theta = \frac{2\sqrt{13}}{13}}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(-\frac{2}{3}\right)^2 = \sec^2\theta$

$1 + \frac{4}{9} = \sec^2\theta$

$\frac{13}{9} = \sec^2\theta$

$\sec\theta = \pm\frac{\sqrt{13}}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{3}{\sqrt{13}}\right)^2 = 1$

$\sin^2\theta + \frac{9}{13} = 1$

$\sin^2\theta = \frac{4}{13}$

$\sin\theta = \pm\frac{2}{\sqrt{13}}$

4) Find $\sec\theta$ and $\sin\theta$ if $\tan\theta = -\frac{5}{7}$ and $\cos\theta > 0$.

$\boxed{\sec\theta = \frac{\sqrt{74}}{7} \quad \sin\theta = -\frac{5\sqrt{74}}{74}}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(-\frac{5}{7}\right)^2 = \sec^2\theta$

$1 + \frac{25}{49} = \sec^2\theta$

$\frac{74}{49} = \sec^2\theta$

$\sec\theta = \pm\frac{\sqrt{74}}{7}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{7}{\sqrt{74}}\right)^2 = 1$

$\sin^2\theta + \frac{49}{74} = 1$

$\sin^2\theta = \frac{25}{74}$

$\sin\theta = \pm\frac{5}{\sqrt{74}}$

5) Find $\sec\theta$ and $\cot\theta$ if $\sin\theta = \frac{4}{7}$ and $\sec\theta > 0$.

$\boxed{\sec\theta = \frac{7\sqrt{33}}{33} \quad \cot\theta = \frac{\sqrt{33}}{4}}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{4}{7}\right)^2 + \cos^2\theta = 1$

$\frac{16}{49} + \cos^2\theta = 1$

$\cos^2\theta = \frac{33}{49}$

$\cos\theta = \pm\frac{\sqrt{33}}{7}$

$\sec\theta = \frac{7}{\sqrt{33}}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{\frac{\sqrt{33}}{7}}{\frac{4}{7}}$

$\cot\theta = \frac{\sqrt{33}}{4}$

6) Find $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{1}{2}$ and $\tan\theta > 0$.

$\boxed{\cos\theta = \frac{\sqrt{3}}{2} \quad \tan\theta = \frac{\sqrt{3}}{3}}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

Verify each identity.

7) $\dfrac{\tan^2 x}{\cos x} = \dfrac{\sec x}{\cot^2 x}$

$$\frac{\frac{1}{\cot^2 x}}{\frac{1}{\sec x}} = \frac{1}{\cot^2 x} \cdot \frac{\sec x}{1}$$

$$\boxed{\frac{\sec x}{\cot^2 x}}$$

8) $\dfrac{\csc x}{1 + \csc x} = \dfrac{1}{1 + \sin x}$

$$\frac{\frac{1}{\sin x}}{1 + \frac{1}{\sin x}} = \frac{\frac{1}{\sin x}}{\frac{\sin x}{\sin x} + \frac{1}{\sin x}} = \frac{\frac{1}{\sin x}}{\frac{\sin x + 1}{\sin x}}$$

$$= \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\sin x + 1} = \boxed{\frac{1}{\sin x + 1}}$$

9) $\cot^2 x - \tan^2 x = \csc^2 x - \sec^2 x$

$* \cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$* 1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\csc^2 x - 1 - (\sec^2 x - 1)$

$\csc^2 x - 1 - \sec^2 x + 1$

$$\boxed{\csc^2 x - \sec^2 x}$$

10) $1 + \csc x\cos^2 x = \dfrac{\sin x + \cos^2 x}{\sin x}$

$$1 + \frac{1}{\sin x} \cdot \frac{\cos^2 x}{1}$$

$$\frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

$$\boxed{\frac{\sin x + \cos^2 x}{\sin x}}$$

11) $-\sin x \sec x = -\tan x$

$$\frac{-\sin x}{1} \cdot \frac{1}{\cos x}$$

$$-\frac{\sin x}{\cos x} = \boxed{-\tan x}$$

12) $\dfrac{\cot x + \sin x}{\csc x} = \cos x + \sin^2 x$

$$\frac{\frac{\cos x}{\sin x} + \frac{\sin x}{1} \cdot \frac{\sin}{\sin}}{\frac{1}{\sin x}}$$

$$\frac{\cos x + \sin^2 x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1} = \boxed{\cos x + \sin^2 x}$$

13) $\sec x + \csc x \cot x = \dfrac{\csc^2 x}{\cos x}$

$$\frac{1}{\cos x} + \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = {}^{\sin^2}\frac{1}{\cos x} + \frac{\cos x}{\sin^2 x}{}^{\cos} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin^2 x}$$

$$= \frac{1}{\cos x \sin^2 x} = \frac{1}{\cos x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos x} \cdot \csc^2 x$$

$$= \boxed{\frac{\csc^2 x}{\cos x}}$$

14) $-\tan x\cos x = \dfrac{\cot^2 x - \csc^2 x}{\csc x}$

$* \cot^2 x + 1 = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\cot^2 x - \csc^2 x = -1$

$-\dfrac{\sin x}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{1} = -\sin x$ $\quad -1(\sin x)$

$= -1$

$\sin x(\cot^2 x - \csc^2 x)$

$\dfrac{1}{\csc x}(\cot^2 x - \csc^2 x) = \boxed{\dfrac{\cot^2 x - \csc^2 x}{\csc x}}$

15) $\csc^2 x\tan^2 x = \tan^2 x + 1$

$\dfrac{1}{\cancel{\sin^2 x}} \cdot \dfrac{\cancel{\sin^2 x}}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x = \boxed{\tan^2 x + 1}$

16) $\tan x \cdot (1 + \cot^2 x) = \dfrac{\csc x}{\cos x}$

$\dfrac{\sin x}{\cos x} \cdot \dfrac{\csc^2 x}{1} = \dfrac{\cancel{\sin x}}{\cos x} \cdot \dfrac{1}{\sin^{\cancel{2}} x} = \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}$

$= \dfrac{1}{\cos x} \cdot \csc x = \boxed{\dfrac{\csc x}{\cos x}}$

17) $\cot^2 x + \sec^2 x\cos^2 x = \dfrac{\csc x}{\sin x}$

$$\frac{\cos^2 x}{\sin^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$$

$$\frac{\cos^2 x}{\sin^2 x} + 1 = \frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \frac{1}{\sin x} \cdot \frac{1}{\sin x}$$

$$= \csc x \cdot \frac{1}{\sin x}$$

$$= \boxed{\frac{\csc x}{\sin x}}$$

18) $\sec^2 x + \csc^2 x = \dfrac{\csc^2 x}{\cos^2 x}$

$$\sin^2 x \; \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \; \cos^2 x = \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{\csc^2 x}{1}$$

$$= \boxed{\frac{\csc^2 x}{\cos^2 x}}$$

Find the exact value of each.

19) $\cos\theta = -\dfrac{3}{5}$ where $\pi \le \theta < \dfrac{3\pi}{2}$

Find $\cos 2\theta$

$\ominus$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$2\left(-\frac{3}{5}\right)^2 - 1$$

$$2\left(\frac{9}{25}\right) - 1$$

$$\frac{18}{25} - \frac{25}{25} = \boxed{\frac{-7}{25}}$$

20) $\tan\theta = \dfrac{4}{3}$ where $0 \le \theta < \dfrac{\pi}{2}$

Find $\tan 2\theta$

$\ominus$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{\frac{9}{9} - \frac{16}{9}}$$

$$= \frac{8}{\cancel{3}} \cdot \frac{\cancel{9}^{3}}{-7} = \boxed{-\frac{24}{7}}$$

21) $\sin\theta = -\frac{12}{13}$ where $\frac{3\pi}{2} \le \theta < 2\pi$

Find $\sin 2\theta$

$x = +5$
$y = -12$
$r = 13$
$x^2 + y^2 = r^2$
$x^2 + 144 = 169$
$x^2 = 25$
$x = \pm 5$

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right)$
$= \boxed{\frac{-120}{169}}$

22) $\cos\theta = -\frac{8}{17}$ where $\pi \le \theta < \frac{3\pi}{2}$

Find $\cos 2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$
$2\left(-\frac{8}{17}\right)^2 - 1$
$2\left(\frac{64}{289}\right) - 1$
$\frac{128}{289} - \frac{289}{289} = \boxed{\frac{-161}{289}}$

23) $\cos\theta = -\frac{3}{5}$ where $\pi \le \theta < \frac{3\pi}{2}$

Find $\cos 2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$
$= 2\left(-\frac{3}{5}\right)^2 - 1$
$2\left(\frac{9}{25}\right) - 1$
$\frac{18}{25} - \frac{25}{25} = \boxed{\frac{-7}{25}}$

24) $\cos\theta = \frac{8}{17}$ where $0 \le \theta < \frac{\pi}{2}$

Find $\sin 2\theta$

$x = 8$
$y = 15$
$r = 17$
$x^2 + y^2 = r^2$
$64 + y^2 = 289$
$y^2 = 225$
$y = 15$

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right)$
$= \boxed{\frac{240}{289}}$

Verify each identity.

25) $\frac{\cos^2 x - \cos 2x}{\csc^2 x} = \sin^4 x$

$\frac{\cos^2 x - (\cos^2 x - \sin^2 x)}{\csc^2 x} = \frac{\sin^2 x}{\csc^2 x} = \sin^2 x \cdot \sin^2 x = \boxed{\sin^4 x}$

26) $1 + \cos 2x + 2\sin x\cos x = 2\cos^2 x + \sin 2x$

$1 + 2\cos^2 x - 1 + \underbrace{2\sin x\cos x}_{\sin 2x}$

$\boxed{2\cos^2 x + \sin 2x}$

27) $\frac{1-\cos 2x}{1+\cos 2x}=\frac{\tan x}{\cot x}$ $\rightarrow$ $\frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos^2 x}$

$$\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)} = \frac{1-1+2\sin^2 x}{1+2\cos^2 x-1} = \frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} = \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\cos x} = \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \boxed{\frac{\tan x}{\cot x}}$$

28) $\underbrace{\tan^2 x} + 2\cos^2 x = \cos 2x + \sec^2 x$

$\sec^2\theta \underbrace{-1 + 2\cos^2 x}$

$\underbrace{2\cos^2 x - 1} + \sec^2 x$

$\boxed{\cos 2x + \sec^2 x}$

29) $\frac{\csc x}{\csc^2 x - 2} = \frac{\sin x}{\cos 2x}$

$$\frac{\frac{1}{\sin x}}{\frac{1}{\sin^2 x}-\frac{2}{1}} = \frac{\frac{1}{\sin x}}{\frac{1-2\sin^2 x}{\sin^2 x}} = \frac{1}{\cancel{\sin x}}\cdot\frac{\sin^2 x}{1-2\sin^2 x}$$

$$= \frac{\sin x}{\underbrace{1-2\sin^2 x}_{\cos 2x}} = \boxed{\frac{\sin x}{\cos 2x}}$$

30) $\dfrac{\sin x}{\tan x} = 1 + \underbrace{\cos 2x}_{} - 2\cos^2 x + \cos x$

$\cancel{1} + \cancel{2\cos^2 x} - \cancel{1} - \cancel{2\cos^2 x} + \cos x$

$= \dfrac{\cos x}{1} \cdot \dfrac{\sin x}{\sin x} = \boxed{\dfrac{\cos x \sin x}{\sin x}}$

$\cot x \cdot \sin x = \boxed{\dfrac{\sin x}{\tan x}}$

31) $2\tan^2 x\cos^2 x = 1 - \cos 2x$

skip

32) $\dfrac{1}{\tan^2 x(1 + \cos 2x)} = \dfrac{1}{2\sin^2 x}$

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