Final Exam Review Solutions

(1) $4x - y + 5 = 0$ use $m = 4$ $\quad y - y_1 = m(x - x_1)$

$y = 4x + 5$ $\quad y - 3 = 4(x + 5)$

$y - 3 = 4x + 20$

$\boxed{y = 4x + 23}$

(2) $f(x) = \frac{2x+1}{x+3}$ denom. cannot be 0

so D: $x \neq -3$

(3) $f(3) = 3 - 5 = \boxed{-2}$

(4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x}{\frac{1}{x}} = x \cdot x = \boxed{x^2;\ x \neq 0}$

(5) $\frac{\text{factors of } 2}{\text{factors of } 18} = \pm \frac{1, 2}{1, 2, 3, 6, 9, 18}$

potential roots are: $\pm 1, 2, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{18}, \frac{2}{3}, \frac{2}{9}$

(6) shift right 4 units and up 5 units

(7) $f(x) = 3^{x+2}$ $\quad x = 3^{y+2}$

$y = 3^{x+2}$ $\quad \log_3 x = y + 2$

$\log_3 x - 2 = y$

$\boxed{f^{-1}(x) = \log_3 x - 2}$

(8) $x = \frac{-b}{2a} = \frac{-2}{2(3)} = -\frac{2}{6} = -\frac{1}{3}$

$y = 3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) - 1 = 3\left(\frac{1}{9}\right) - \frac{2}{3} - 1 = \frac{1}{3} - \frac{2}{3} - \frac{3}{3} = -\frac{4}{3}$

vertex: $\left(-\frac{1}{3}, -\frac{4}{3}\right)$

(9) $g(f(x)) = 1 - \frac{1}{x} - 1 = \boxed{-\frac{1}{x}}$

(10) $2 - 3\ln(x+1) = 8$

$-3\ln(x+1) = 6$

$\ln(x+1) = -2$

$e^{-2} = x+1$

$e^{-2} - 1 = x$

$\boxed{x = -.865}$

(11) $2^{x^2} = 4^x$

$2^{x^2} = 2^{2x}$

$x^2 = 2x$

$x^2 - 2x = 0$

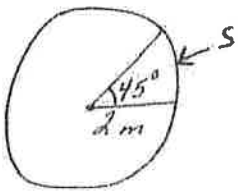
$x(x-2)$

$\boxed{x = 0}$

$\boxed{x = 2}$

(12) $\log_3 \pi = \frac{\log \pi}{\log 3} = \boxed{1.042}$

(13)



$45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$

$s = r\theta = 2\left(\frac{\pi}{4}\right) =$

$\frac{\pi}{2} = \boxed{1.57 \text{ meters}}$

(14) $\csc\left(-\frac{5\pi}{4}\right) = \csc(-225°)$ * same as $\csc 135°$

$\boxed{\sqrt{2}}$

(15) $\sin\theta = \frac{-2}{5}$ → opp, hyp

$a^2 + b^2 = c^2$

$a^2 + (-2)^2 = 5^2$

$a^2 + 4 = 25$

$a^2 = 21 \quad a = \sqrt{21}$

$\sec\theta = \frac{5}{\sqrt{21}}$ pos in Q4

$\sec\theta = \frac{5\sqrt{21}}{21}$

OR $\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{2}{5}\right)^2 + \cos^2\theta = 1$

$\frac{4}{25} + \cos^2\theta = 1$

$\cos^2\theta = \frac{21}{25}$

$\cos\theta = \frac{\sqrt{21}}{5}$

$\sec\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

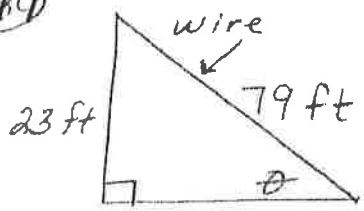
(16) $\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

(17) $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{2}{25}\right) = \frac{25}{25} - \frac{18}{25} = \frac{7}{25}$

(16) $-\frac{22\pi}{3} + 2\pi = -\frac{22\pi}{3} + \frac{6\pi}{3} = \frac{-16\pi}{3}$ one of many answers

(17)



$\sin\theta = \frac{23}{79}$

$\theta = 16.926°$

(18) $\frac{210}{1}\cdot\frac{\pi}{180} = \frac{7\pi}{6}$

(19) $(\cos\alpha + \sin\alpha)(\cos\alpha + \sin\alpha)$

$\cos^2\alpha + 2\sin\alpha\cos\alpha + \sin^2\alpha$

$\boxed{1 + 2\sin\alpha\cos\alpha}$

(20) Use law of cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 4.7^2 + 10.21^2 - 2(4.7)(10.21)\cos 105.3$

$c^2 = 151.659$

$\boxed{c = 12.315}$

(21) $\sqrt{-(-1)} + (1+1)$

$1 + 2$

$\boxed{3}$

(22) $\frac{\sin 30}{10} = \frac{\sin 40}{x}$

$x\sin 30 = 10\sin 40$

$\boxed{x = 12.856}$

(23) $f(x) = \frac{\sqrt{x}}{x-1}$ denom cannot $= 0$

radicand must be non-negative

numerator: $x \geq 0$

denom: $x \neq 1$

$\boxed{D: [0,1) \cup (1,\infty)}$

(24) $y = a(x-h)^2 + k$ → equation of a parabola

$-1 = a(0-2)^2 + 3$

$-1 = 4a + 3$

$-4 = 4a$

$a = -1$

Point $(\overset{x}{0}, \overset{y}{-1})$

Vertex $(\underset{h}{2}, \underset{k}{3})$

$\boxed{y = -(x-2)^2 + 3}$

(25) $f(g(x)) = (\sqrt{x-1})^2 + \sqrt{x-1}$

$= \boxed{x-1+\sqrt{x-1}}$

(26) find $f^{-1}(x)$ first

$f(x) = x^3 - 1$

$y = x^3 - 1$

$x = y^3 - 1$

$x+1 = y^3$

$y = \sqrt[3]{x+1}$

$f^{-1}(x) = \sqrt[3]{x+1}$

$f^{-1}(26) = \sqrt[3]{26+1}$

$= \sqrt[3]{27}$

$\boxed{= 3}$

(27) $y = \frac{x+3}{2}$

$x = \frac{y+3}{2}$

$2x = y+3$

$y = 2x-3$

$\boxed{f^{-1}(x) = 2x-3}$

(28) $x = \frac{-b}{2a} = \frac{-(-14)}{2(1)} = \frac{14}{2} = 7$ $\boxed{x = 7}$

(29) factors of 12 / factors of 1 $\boxed{\pm 1, 2, 3, 4, 6, 12}$

(30) $\frac{1}{3}\log_2(x+6) = \log_2 3$

$\log_2(x+6)^{\frac{1}{3}} = \log_2 3$

$(\sqrt[3]{x+6})^3 = (3)^3$ → $x+6 = 27$

$\boxed{x = 21}$

(31) $\log_3 10 = \frac{\log 10}{\log 3} = \boxed{2.096}$

(32) $2\log x + \frac{1}{2}\log y - 3\log z$

$\log x^2 + \log y^{1/2} - \log z^3 = \boxed{\log \frac{x^2 y^{1/2}}{z^3}}$

(33) $x^3 = 27 \rightarrow \boxed{\log_x 27 = 3}$

(34) $2^{x+2} = 16^x$

$2^{x+2} = 2^{4x}$

$x + 2 = 4x$

$3x = 2$

$x = 2/3$

(35) Given: $s = \pi$, $r = 4$

$s = r\theta$

$\pi = 4\theta$

$\theta = \frac{\pi}{4}$ $\boxed{\theta = 45^\circ}$

(36) $\sin x = \frac{1}{2}$ use the unit circle
x only in Q1 + Q2

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

(37) $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 3^2 + 2^2 - 2(3)(2)\cos 45$

$c^2 = 9 + 4 - 12\cos 45$

$c^2 = 13 - 12\cos$

$c^2 = 4.5147$

$\boxed{c = 2.123}$

(38) $\frac{\sin 70}{b} = \frac{\sin 50}{5}$

$5\sin 70 = b\sin 50$

$\boxed{b = 6.133}$

(39) $\frac{\sec t \tan t}{\tan^2 t + 1} = \frac{\frac{1}{\cos} \cdot \frac{\sin}{\cos}}{\sec^2} = \frac{\frac{\sin}{\cos^2}}{\frac{1}{\cos^2}} = \frac{\sin}{\cos^2} \cdot \frac{\cos^2}{1}$

$\boxed{= \sin t}$

(40) $\sec t \cot t - \cot t \cos t$

$\frac{1}{\cos} \cdot \frac{\cos}{\sin} - \frac{\cos}{\sin} \cdot \frac{\cos}{1}$

$\frac{1}{\sin} - \frac{\cos^2}{\sin} = \frac{1 - \cos^2}{\sin} = \frac{\sin^2}{\sin} = \boxed{\sin t}$

(41) $$\frac{\cot^2 \cos^2}{\cot^2 - \cos^2} = \frac{\frac{\cos^2}{\sin^2} \cdot \frac{\cos^2}{1}}{\frac{\cos^2}{\sin^2} - \frac{\cos^2}{1}} = \frac{\frac{\cos^4}{\sin^2}}{\frac{\cos^2 - \cos^2 \sin^2}{\sin^2}}$$

$$= \frac{\cos^4}{\sin^2} \cdot \frac{\sin^2}{\cos^2 - \cos^2 \sin^2} = \frac{\cos^4}{\cos^2(1 - \sin^2)} = \frac{\cos^4}{\cos^2/\cos^2} = \boxed{1}$$

(42) $$-2\cos x = \sqrt{2}$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{3\pi}{4} + 2\pi n \qquad x = \frac{5\pi}{4} + 2\pi n}$$

(43) $$2\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{\pi}{4} + 2\pi n \qquad x = \frac{3\pi}{4} + 2\pi n}$$

(44) $$2\cos^2 x - 3\sin x = 0$$

$$2(1 - \sin^2 x) - 3\sin x = 0$$

$$2 - 2\sin^2 x - 3\sin x = 0$$

$$-2\sin^2 x - 3\sin x + 2 = 0$$

$$(-\sin x - 2)(2\sin x - 1) = 0$$

$$-\sin x - 2 = 0 \qquad 2\sin x - 1 = 0$$

$$-\sin x = 2 \qquad 2\sin x = 1$$

$$\sin x = -2 \qquad \sin x = \frac{1}{2}$$

$\emptyset$

$$\boxed{x = \frac{\pi}{6} + 2\pi n \qquad x = \frac{5\pi}{6} + 2\pi n}$$

(45) $3\tan^3 x + 1 = 3\tan^2 x + \tan x$

set = 0 then factor by grouping

$3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$

$3\tan^2 x(\tan x - 1) - 1(\tan x - 1) = 0$

$(3\tan^2 x - 1)(\tan x - 1) = 0$

$3\tan^2 x - 1 = 0$ $\quad$ $\tan x - 1 = 0$

$3\tan^2 x = 1$ $\quad$ $\tan x = 1$

$\tan^2 x = \frac{1}{3}$ $\quad$ $x = \frac{\pi}{4} + n\pi$

$\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6} + n\pi$

(46) $\tan x = -3$ $\quad$ * $\tan^{-1}$ in calculator

$x = -71.565$