

# 15-1 Skills Practice

## Trigonometric Identities

Find the exact value of each expression if  $0^\circ < \theta < 90^\circ$ .

1. If  $\tan \theta = 1$ , find  $\sec \theta$ .

$\sqrt{2}$

2. If  $\tan \theta = \frac{1}{2}$ , find  $\cos \theta$ .

$\frac{2\sqrt{5}}{5}$

3. If  $\sec \theta = 2$ , find  $\cos \theta$ .

$\frac{1}{2}$

4. If  $\cos \theta = \frac{8}{17}$ , find  $\csc \theta$ .

$\frac{17}{15}$

Find the exact value of each expression if  $90^\circ < \theta < 180^\circ$ .

5. If  $\cos \theta = -\frac{4}{5}$ , find  $\sin \theta$ .

$\frac{3}{5}$

6. If  $\cot \theta = -\frac{3}{2}$ , find  $\cos \theta$ .

$-\frac{3\sqrt{13}}{13}$

Find the exact value of each expression if  $180^\circ < \theta < 270^\circ$ .

7. If  $\tan \theta = 1$ , find  $\cos \theta$ .

$-\frac{\sqrt{2}}{2}$

8. If  $\sin \theta = -\frac{\sqrt{2}}{2}$ , find  $\tan \theta$ .

1

9. If  $\csc \theta = -2$ , find  $\cos \theta$ .

$-\frac{\sqrt{3}}{2}$

10. If  $\cos \theta = -\frac{2\sqrt{5}}{5}$ , find  $\tan \theta$ .

$\frac{1}{2}$

11. If  $\csc \theta = -2$ , find  $\cot \theta$ .

$\sqrt{3}$

12. If  $\sin \theta = -\frac{5}{13}$ , find  $\tan \theta$ .

$\frac{5}{12}$

Simplify each expression.

13.  $\sin \theta \sec \theta$   $\tan \theta$

14.  $\csc \theta \sin \theta$  1

15.  $\cot \theta \sec \theta$   $\csc \theta$

16.  $\frac{\cos \theta}{\sec \theta}$   $\cos^2 \theta$

17.  $\tan \theta + \cot \theta$   $\frac{1}{\cos \theta \sin \theta}$

18.  $\csc \theta \tan \theta - \tan \theta \sin \theta$   $\cos \theta$

19.  $\frac{1 - \sin^2 \theta}{\sin \theta + 1}$   $1 - \sin \theta$

20.  $\csc \theta + \cot \theta$   $\frac{1 + \cos \theta}{\sin \theta}$

21.  $\frac{\sin^2 \theta + \cos^2 \theta}{1 - \cos^2 \theta}$   $\csc^2 \theta$

22.  $1 + \frac{\tan^2 \theta}{1 + \sec \theta}$   $\sec \theta$

# 17-1 Skills Practice

$$0^\circ < \theta < 90^\circ$$

$$1) \tan \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$\sec \theta = \pm \sqrt{2} \quad 1 + 1^2 = \sec^2 \theta$$

$$2 = \sec^2 \theta$$

$$\sec \theta = \pm \sqrt{2}$$

$$2) \tan \theta = \frac{1}{2} \quad \text{find } \sec \theta \text{ then flip it!}$$

$$\cos \theta = \pm \frac{2\sqrt{5}}{5} \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \frac{1}{4} = \sec^2 \theta$$

$$\frac{5}{4} = \sec^2 \theta$$

$$\pm \frac{\sqrt{5}}{2} = \sec \theta$$

$$\cos \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$3) \sec \theta = 2$$

$$\cos \theta = \pm \frac{1}{2} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$2 = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{2}$$

$$4) \cos \theta = \frac{8}{17} \quad \text{need to find } \sin \theta \text{ then flip it!}$$

$$\csc \theta = \pm \frac{17}{15} \quad \cos^2 \theta + \sin^2 \theta = 1$$

$$\left(\frac{8}{17}\right)^2 + \sin^2 \theta = 1$$

$$\frac{64}{289} + \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{225}{289}$$

$$\sin \theta = \pm \frac{15}{17}$$

$90^\circ < \theta < 180^\circ$  Quadrant II

5)  $\cos \theta = -\frac{4}{5}$

$\sin \theta = +\frac{3}{5}$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\left(-\frac{4}{5}\right)^2 + \sin^2 \theta = 1$$

$$\frac{16}{25} + \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\sin \theta = \pm \frac{3}{5}$$

sin pos in QII

6)  $\cot \theta = -\frac{3}{2}$

$\cos \theta = -\frac{3\sqrt{13}}{13}$

• flip  $\cot \theta$  so  $\tan \theta = -\frac{2}{3}$

• find  $\sec \theta$  then flip for  $\cos \theta$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \left(-\frac{2}{3}\right)^2 = \sec^2 \theta$$

$$\frac{9}{9} + \frac{4}{9} = \sec^2 \theta$$

$$\frac{13}{9} = \sec^2 \theta$$

$$\pm \frac{\sqrt{13}}{3} = \sec \theta$$

$$\cos \theta = \pm \frac{3}{\sqrt{13}} = \pm \frac{3\sqrt{13}}{13}$$

cos neg in QII

$$180^\circ < \theta < 270^\circ \quad \text{Quadrant III}$$

$$8) \sin \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = \underline{+1}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\tan \theta = -1$$

find  $\cos \theta$  first!

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta + \left(-\frac{\sqrt{2}}{2}\right)^2 = 1$$

$$\cos^2 \theta + \frac{2}{4} = 1$$

$$\cos^2 \theta = \frac{2}{4}$$

$$\cos \theta = \pm \frac{\sqrt{2}}{2}$$

tan pos in QIII

$$10) \cos \theta = \frac{-2\sqrt{5}}{5}$$

$$\tan \theta = \underline{+\frac{1}{2}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{\frac{\sqrt{5}}{5}}{\frac{-2\sqrt{5}}{5}} = \frac{\sqrt{5}}{5} \cdot \frac{5}{-2\sqrt{5}} = -\frac{1}{2}$$

find  $\sin \theta$  first

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\left(\frac{-2\sqrt{5}}{5}\right)^2 + \sin^2 \theta = 1$$

$$\frac{20}{25} + \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{5}{25}$$

$$\sin \theta = \pm \frac{\sqrt{5}}{5}$$

tan pos in QIII

$180 < \theta < 270$  Quadrant III

7)  $\tan \theta = 1$   
 $\cos \theta = -\frac{\sqrt{2}}{2}$

$$1 + \tan^2 \theta = \sec^2 \theta$$
$$1 + 1^2 = \sec^2 \theta$$
$$\sec \theta = \pm \sqrt{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$
$$\cos \theta = \frac{\sqrt{2}}{2}$$

neg in III

9)  $\csc \theta = -2$   
 $\cos \theta = -\frac{\sqrt{3}}{2}$

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\left(-\frac{1}{2}\right)^2 + \cos^2 \theta = 1$$

neg in III

$$\frac{1}{4} + \cos^2 \theta = 1$$
$$\cos^2 \theta = \frac{3}{4}$$
$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

11)  $\csc \theta = -2$   
 $\cot \theta = \sqrt{3}$

$$1 + \cot^2 \theta = \csc^2 \theta$$
$$1 + \cot^2 \theta = (-2)^2$$
$$\cot^2 \theta = 3$$
$$\cot \theta = \pm \sqrt{3}$$

cot pos in III

12)  $\sin \theta = \frac{-5}{13}$   
 $\tan \theta = \frac{5}{12}$

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\left(\frac{-5}{13}\right)^2 + \cos^2 \theta = 1$$
$$\frac{25}{169} + \cos^2 \theta = 1$$
$$\cos^2 \theta = \frac{144}{169}$$
$$\cos \theta = \pm \frac{12}{13}$$

tan pos in III

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\tan \theta = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$13) \csc \theta \sec \theta$$

$$\frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta}$$

$$\frac{\sin \theta}{\cos \theta} = \boxed{\tan \theta}$$

$$15) \cot \theta \sec \theta$$

$$\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta}$$

$$\frac{1}{\sin \theta} = \boxed{\csc \theta}$$

$$17) \tan \theta + \cot \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{(\cos \theta)(\sin \theta)}$$

$$\frac{1}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\boxed{\sec \theta \csc \theta}$$

$$20) \csc \theta + \cot \theta$$

$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\boxed{\frac{1 + \cos \theta}{\sin \theta}}$$

$$14) \csc \theta \sin \theta$$

$$\frac{1}{\sin \theta} \cdot \sin \theta = \boxed{1}$$

$$16) \frac{\cos \theta}{\sec \theta} = \frac{\cos \theta}{\frac{1}{\cos^2 \theta}}$$

$$= \cos \theta \cdot \frac{\cos^2 \theta}{1} = \boxed{\cos^3 \theta}$$

$$18) \csc \theta \tan \theta - \tan \theta \sin \theta$$

$$\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{1}$$

$$\frac{1}{\cos \theta} - \frac{\sin^2 \theta}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta}$$

$$\boxed{\cos \theta}$$

$$19) \frac{1 - \sin^2 \theta}{\sin \theta + 1}$$

← factor!

$$\frac{(1 - \sin \theta)(1 + \sin \theta)}{\cancel{(1 + \sin \theta)}}$$

$$\boxed{1 - \sin \theta}$$

$$21) \frac{\sin^2 \theta + \cos^2 \theta}{1 - \cos^2 \theta} = \frac{1}{\sin^2 \theta} = \boxed{\csc^2 \theta}$$

---

$$22) 1 + \frac{\tan^2 \theta}{1 + \sec \theta}$$

$$1 + \frac{\sec^2 \theta - 1}{1 + \sec \theta}$$

$$1 + \frac{(\sec \theta + 1)(\sec \theta - 1)}{1 + \sec \theta}$$

$$1 + \sec \theta - 1 = \boxed{\sec \theta}$$