12-5 Probability of Compound Events

A **simple event** consists of just one outcome.

Example: Rolling a 3 on a die

A **compound event** is made up of two or more simple events.

Example: Rolling a 3 on a die and flipping heads on a coin.

Two events are called **independent events** if the outcome of one does not affect the outcome of the other.

If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.

$P(A \text{ and } B) = P(A) \cdot P(B)$

* and on then means multiply

Example: A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles and 2 green marbles. A marble is selected, replaced, and a second marble is selected.

1. Find the probability of selecting a black marble, then a yellow marble.

21 total marbles

$\frac{6 \text{ black}}{21 \text{ total}} \cdot \frac{4 \text{ yellow}}{21 \text{ total}}$

$\frac{24}{441} = \boxed{\frac{8}{147}}$

2. Find the probability of selecting a blue marble, then a green marble.

$\frac{9 \text{ blue}}{21 \text{ total}} \cdot \frac{2 \text{ green}}{21 \text{ total}} = \frac{18}{441} = \boxed{\frac{2}{49}}$

3. Find the probability of selecting a marble that is **not black**, then a yellow marble.

21 total
− 6 black
15 not black

$\frac{15}{21} \cdot \frac{4 \text{ yellow}}{21 \text{ total}} = \frac{60}{441} = \boxed{\frac{20}{147}}$

Two events are called **dependent events** if the outcome of one affects the outcome of the other.

If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

P(A and B) = P(A) · P(B following A)

Example: Cynthia randomly draws three cards from a standard deck one at a time without replacement. Find the probability that the cards are drawn in the given order.

1. P(diamond, spade, diamond)

$\frac{13 \text{ diamonds}}{52 \text{ total cards}} \cdot \frac{13 \text{ spades}}{51 \text{ total left}} \cdot \frac{12 \text{ diamonds left}}{50 \text{ total left}}$

$\frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} = \boxed{\frac{13}{850}}$

2. P(four, four, not a jack)

$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{46}{51} = \frac{23}{5525}$

3. P(two, five, not a five)

$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{47}{50} = \boxed{\frac{94}{16575}}$

12
13 heart
13 diamonds
13 spades
13 clubs
52 total
51
50

4. P(heart, not a heart, heart)

$\frac{13}{52} \cdot \frac{39}{51} \cdot \frac{12}{50} = \boxed{\frac{39}{850}}$

Two events are called **mutually exclusive events** if they are events that cannot occur at the same time.

If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

P(A or B) = P(A) + P(B)

Example: A die is being rolled. Find each probability.

1. P(3 or 5)

or means add!

$P(3) + P(5)$

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

2. P(at least 4)

$P(4 \text{ or } 5 \text{ or } 6)$

$P(4) + P(5) + P(6)$

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \boxed{\frac{1}{2}}$

3. P(less than 3)

$P(1 \text{ or } 2)$

$P(1) + P(2)$

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

4. P(even)

$P(2 \text{ or } 4 \text{ or } 6)$

$P(2) + P(4) + P(6)$

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \boxed{\frac{1}{2}}$

Two events are called **not mutually exclusive events** if they are events that can possibly occur at the same time.

If two events, A and B, are not mutually exclusive, then the probability that either A or B occur is the sum of their probabilities decreased by the probability of both occurring.

P(A or B) = P(A) + P(B) – P(A and B)

Examples: 1. Of 240 girls, 176 are on the Honor Roll, 48 play sports, and 36 are on the Honor Roll and play sports. What is the probability that a randomly selected student plays sports or is on the Honor Roll?

$P(\text{Sports}) + P(\text{Honor Roll}) - P(\text{Sports} + \text{HR})$

$\frac{48}{240} + \frac{176}{240} - \frac{36}{240} = \frac{188}{240} = \boxed{\frac{47}{60}}$

2. Out of 5200 households surveyed, 2107 had a dog, 807 had a cat, and 303 had both a dog and a cat. What is the probability that a randomly selected household has a dog or a cat?

$P(\text{dog}) + P(\text{cat}) - P(\text{dog} + \text{cat})$

$\frac{2107}{5200} + \frac{807}{5200} - \frac{303}{5200} = \frac{2611}{5200}$

Examples:

*Determine whether the events are **independent or dependent.** Then find the probability.*

1. A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Leah randomly chooses 2 items for the child she is babysitting. What is the probability that she chose two stuffed animals as the first two choices?

Dependent

23 possibilities

$\frac{8 \text{ stuffed}}{23 \text{ total}} \cdot \frac{7 \text{ stuffed}}{22 \text{ total}} = \frac{56}{506}$

$\boxed{\frac{28}{253}}$

2. A basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Lucas randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then an apple?

dependent

20 pieces of fruit

$\frac{5 \text{ bananas}}{20 \text{ total}} \cdot \frac{6 \text{ apples}}{19 \text{ total}} = \frac{30}{380} = \boxed{\frac{3}{38}}$

3. Tucker has 4 quarters, 3 dimes, and 2 nickels in his pocket. Tucker randomly picks two coins out of his pocket. What is the probability that he did **not** choose a dime either time, if he replaced the first coin before choosing a second coin?

Independent

9 coins total

$\frac{6}{9} \cdot \frac{6}{9} = \frac{36}{81} = \boxed{\frac{4}{9}}$

4. If a coin is tossed four times, what is the probability of getting tails all four times?

Independent

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{16}}$

5. A die is rolled twice. What is the probability of rolling two different numbers?

11, 12, 13, 14, 15, 16
21, ~~22~~, 23, 24, 25, 26
31, 32, ~~33~~, 34, 35, 36
41, 42, 43, ~~44~~, 45, 46
51, 52, 53, 54, ~~55~~, 56
61, 62, 63, 64, 65, ~~66~~

36 possible outcomes

$\frac{30}{36} = \boxed{\frac{5}{6}}$

6. A card is drawn from a standard deck of playing cards. Once a card is drawn, it is replaced. Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.

a. P(two or queen) mutually exclusive

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$

b. P(diamond or heart) mutually exclusive

$\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \boxed{\frac{1}{2}}$

c. P(seven or club) Not mutually Exclusive

P(7) + P(club) − P(7 of clubs)

$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

d. P(spade or ace) Not mutually exclusive

P(Spade) + P(Ace) − P(Ace of Spades)

$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

12-5 Practice

1. A bag contains 3 red, 4 blue, and 6 yellow marbles. One marble is selected at a time, and once a marble is selected, it is not replaced. Find each probability.

13 total marbles

a. P(2 yellow) $\frac{6}{13} \cdot \frac{5}{12} = \boxed{\frac{5}{26}}$

b. P(red, yellow) $\frac{3}{13} \cdot \frac{6}{12} = \frac{18}{156} = \boxed{\frac{3}{26}}$

c. P(blue, red, yellow)

$$\frac{4}{13} \cdot \frac{3}{12} \cdot \frac{6}{11} = \frac{72}{1716} = \boxed{\frac{6}{143}}$$

2. Destiny has two red socks and two white socks in a drawer. What is the probability of picking a red sock and a white sock in that order if the first sock is not replaced?

4 socks

P(red, white)

$$\frac{2}{4} \cdot \frac{2}{3} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

3. Spencer drops a penny in a pond, and then he drops a nickel in the pond. What is the probability that both coins will land with tails showing?

$$\frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$$

4. A die is rolled and a quarter is dropped. Find the probability of rolling a two and showing a tail.

rolling a 2 · showing a tail

mult.

$$\frac{1}{6} \cdot \frac{1}{2} = \boxed{\frac{1}{12}}$$

5. A card is drawn from a standard decks of playing cards. Find each probability.

a. P(jack or red)

$P(\text{Jack}) + P(\text{red}) - P(\text{Red Jack})$

$\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \boxed{\frac{7}{13}}$

b. P(red or black)

$P(\text{red}) + P(\text{black})$

$\frac{26}{52} + \frac{26}{52} = \frac{52}{52} = \boxed{1}$

c. P(jack or club)

$P(\text{Jack}) + P(\text{club}) - P(\text{Jack of clubs})$

$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

d. P(queen or less than 3)

$P(Q) + P(2)$

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$

e. P(5 or 6)

$P(5) + P(6)$

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$

f. P(diamond or spade)

$P(\diamondsuit) + P(\spadesuit)$

$\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \boxed{\frac{1}{2}}$

6. In a math class, 12 out of 15 girls are 14 years old and 14 out of 17 boys are 14 years old. What is the probability of selecting a girl or a 14-year old from this class?

15 girls
17 boys
32 students

$P(G) + P(14) - P(14 \text{ and girl})$

$\frac{15}{32} + \frac{26}{32} - \frac{12}{32} = \boxed{\frac{29}{32}}$

7. The forecast predicts a 40% chance of snow on Tuesday and a 60% chance of snow on Wednesday. If these probabilities are independent, what is the chance it will rain on both days?

$(.40 \times .60) = .24$

24% chance

8. Travis places his favorite recipes in a bag for 4 pasta dishes, 5 casseroles, 3 types of chili, and 8 desserts.

a. If Travis choses one recipe at random, what is the probability that he selects a pasta dish or a casserole?

20 recipes

$P(\text{pasta or casserole})$

$P(\text{pasta}) + P(\text{casserole})$

$\frac{4}{20} + \frac{5}{20} = \boxed{\frac{9}{20}}$

b. If Travis chooses one recipe at random, what is the probability that he does *not* select a dessert?

20 total
− 8 dessert
12 not dessert

$\frac{12}{20} = \boxed{\frac{3}{5}}$

c. If Travis chooses two recipes at random <u>*without replacement*</u>, what is the probability that the first recipe he selects is a casserole and the second recipe he selects is a dessert?

$\frac{5}{20} \cdot \frac{8}{19} = \frac{40}{380} = \boxed{\frac{2}{19}}$

9. Morgan performs a magic trick in which she holds a standard deck of cards and has each of three people randomly choose a card from the deck. Each person keeps his or her card as the next person draws. What is the probability that all three people will draw a heart?

$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1716}{132600} = \frac{11}{850}$

10. Tiles numbered 1 through 20 are placed in a box. Tiles numbered 11 through 30 are placed in a second box. The first tile is randomly drawn from the first box. The second tile is randomly drawn from the second box. Find each probability.

20 tiles

1-20

20 tiles

11-30

a. P(both are greater than 15)

$$\frac{5}{20} \cdot \frac{15}{20} = \frac{75}{400} = \boxed{\frac{3}{16}}$$

b. The first tile is odd and the second tile is less than 25.

$$\frac{10}{20} \cdot \frac{14}{20} = \frac{140}{200} = \boxed{\frac{7}{10}}$$

(odd) (<25)

c. The first tile is a multiple of 6 and the second tile is a multiple of 4.

mult of 6

6, 12, 18, ~~24~~

mult of 4

~~4~~, 8, 12, 16, 20, 24, 28

$$\frac{3}{20} \cdot \frac{5}{20} = \frac{15}{400} = \boxed{\frac{3}{80}}$$

d. The first tile is less than 15 and the second tile is even or greater than 25.

$$\frac{14}{20} \cdot \frac{12}{25} = \frac{168}{500} = \frac{42}{125}$$

Second tile

$P(\text{even}) + P(>25) - P(\text{even} + >25)$

$$\frac{10}{20} + \frac{5}{25} - \frac{3}{25} = \frac{12}{25}$$