12-5 Probability of Compound Events

A simple event consists of just one outcome.

Example: Rolling a 3 on a die

A compound event is made up of two or more simple events.

Example: Rolling a 3 on a die and flipping heads on a coin.

Two events are called independent events if the outcome of one does not affect the outcome of the other.

If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.

 $P(A \text{ and } B) = P(A) \cdot P(B)$

Example:

A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles and 2 green marbles. A marble is selected, replaced, and a second marble is selected.

1. Find the probability of selecting a black marble, then a yellow marble.

21 total 6 black 4 yellow 21 total

441 = 147

2. Find the probability of selecting a blue marble, then a green marble.

9 6/4e . 3 green = 18 = 2/49/

3. Find the probability of selecting a marble that is not black, then a yellow marble.

- 6 black 15 not black

15 . 4 gellow = 60 = 20 7

Two events are called dependent events if the outcome of one affects the outcome of the other.

If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

 $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example:

Cynthia randomly draws three cards from a standard deck one at a time without replacement. Find the probability that the cards are drawn in the given order.

1. P(diamond, spade, diamond)

$$\frac{13}{50} \cdot \frac{13}{51} \cdot \frac{12}{50} = \begin{bmatrix} 13\\ 850 \end{bmatrix}$$

2. P(four, four, not a jack)

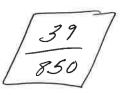
$$\frac{4}{58} \cdot \frac{1}{51} \cdot \frac{1}{51} = \frac{1}{5535}$$

3. P(two, five, not a five)

$$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{47}{50} = \boxed{\frac{94}{16575}}$$

18 heart
13 dramonds $\frac{1}{13}$ 4. P(heart, not a heart, heart)
13 spades $\frac{13}{57}$ 13 clubs $\frac{39}{57}$ $\frac{12}{50}$ $\frac{39}{850}$

$$\frac{39}{51} \cdot \frac{12}{50} =$$



Two events are called mutually exclusive events if they are events that cannot occur at the same time.

If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$

Example:

A die is being rolled. Find each probability.

1. P(3 or 5)

$$P(3) + P(5)$$
 $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

2. P(at least 4)

$$P(4 \text{ or } 5 \text{ or } 6)$$
 $P(4) + P(5) + P(6)$
 $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

3. P(less than 3)

$$P(10r a)$$

$$P(1)+P(a)$$

$$\frac{2}{6}+\frac{3}{6}=\frac{3}{6}$$

4. P(even)

Two events are called **not mutually exclusive events** if they are events that can possibly occur at the same time.

If two events, A and B, are not mutually exclusive, then the probability that either A or B occur is the sum of their probabilities decreased by the probability of both occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Examples: 1. Of 240 girls, 176 are on the Honor Roll, 48 play sports, and 36 are on the Honor Roll and play sports. What is the probability that a randomly selected student plays sports or is on the Honor Roll?

$$P(Sports) + P(HonorRoll) - P(Sports + HR)$$

 $\frac{48}{240} + \frac{176}{240} - \frac{36}{240} = \frac{188}{240} = \frac{47}{60}$

2. Out of 5200 households surveyed, 2107 had a dog, 807 had a cat, and 303 had both a dog and a cat. What is the probability that a randomly selected household has a dog or a cat?

$$\frac{\rho(dog) + \rho(cat) - \rho(dog + cat)}{2107 + 807 - \frac{303}{5200} = \frac{361}{5200}$$

Examples:

Determine whether the events are independent or dependent. Then find the probability.

1. A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Leah randomly chooses 2 items for the child she is babysitting. What is the probability that she chose two stuffed animals as the first two choices?

Dependent $\frac{8 \text{ stuffed}}{8 \text{ stuffed}}, \frac{1 \text{ stuffed}}{32 \text{ total}} = \frac{56}{506}$ $\frac{38}{353}$

2. A basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Lucas randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then an apple?

dependent 20 pieces of fruit $\frac{5 \text{ bananas}}{30 \text{ total}} \cdot \frac{6 \text{ apples}}{19 \text{ total}} = \frac{30}{380} = \boxed{\frac{3}{38}}$

3. Tucker has 4 quarters, 3 dimes, and 2 nickels in his pocket. Tucker randomly picks two coins out of his pocket. What is the probability that he did **not** choose a dime either time, if he replaced the first coin before choosing a second coin?

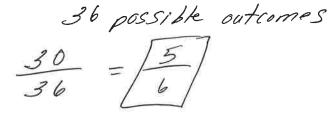
Independent 9 coins total $\frac{b}{9} \cdot \frac{b}{9} = \frac{3b}{81} = \boxed{\frac{4}{9}}$

4. If a coin is tossed four times, what is the probability of getting tails all four times?

Independent $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{1}$

5. A die is rolled twice. What is the probability of rolling two different numbers?

11, 12, 13, 14, 15, 16
21, 28, 23, 24, 25, 26
31, 32, 33, 34, 35, 36
41, 42, 43, 44, 45, 46
51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 46



A card is drawn from a standard deck of playing cards. Once a card is drawn, it is replaced.
Determine whether the events are mutually exclusive or not mutually exclusive. Then find the
probability.

a. P(two or queen) Mutually exclusive $\frac{4}{53} + \frac{4}{53} = \frac{8}{53} = \boxed{13}$

b. P(diamond or heart) mutually exclusive $\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{12}{2}$

c. P(seven or club) Not mutually Exclusive P(7) + P(c/ub) - P(7) + C(ubs) $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{1}{13}$ d. P(spade or ace) Not mutually exclusive $P(5) = \frac{1}{52} + \frac{1}{52} - \frac{1}{52} = \frac{1}{52} = \frac{1}{13}$ $P(5) = \frac{1}{52} + \frac{1}{52} - \frac{1}{52} = \frac{1}{13}$

12-5 Practice

 A bag contains 3 red, 4 blue, and 6 yellow marbles. One marble is selected at a time, and once a marble is selected, it is not replaced. Find each probability.



a. $P(2 \text{ yellow}) = \frac{6}{13} \cdot \frac{5}{13} = \boxed{\frac{5}{36}}$

b. P(red, yellow)
$$\frac{3}{13} \cdot \frac{6}{13} = \frac{18}{156} = \frac{3}{36}$$

c. P(blue, red, yellow)
$$\frac{4}{13} \cdot \frac{3}{13} \cdot \frac{1}{11} = \frac{72}{1716} = \boxed{\frac{6}{143}}$$

2. Destiny has two red socks and two white socks in a drawer. What is the probability of picking a red sock and a white sock in that order if the first sock is not replaced?

$$\frac{4 \operatorname{socks}}{\frac{2}{4} \cdot \frac{3}{3}} = \frac{4}{13} = \begin{bmatrix} \frac{1}{3} \end{bmatrix}$$

3. Spencer drops a penny in a pond, and then he drops a nickel in the pond. What is the probability that both coins will land with tails showing?

4. A die is rolled and a quarter is dropped. Find the probability of rolling a two and showing a tail.



5. A card is drawn from a standard decks of playing cards. Find each probability.

$$P(Jack) + P(red) - P/Red Jack)$$

 $\frac{4}{52} + \frac{36}{53} - \frac{2}{53} = \frac{38}{53} = \boxed{\frac{7}{13}}$

P(red or black)

$$\frac{p(red) + p(black)}{\frac{3b}{5a} + \frac{2b}{5a} = \frac{52}{5a} = \boxed{11}$$

P(jack or club)

$$f(Jack) + f(c/4b) - f(Jack) + f(dubs) - f(dubs) + f(du$$

P(queen or less than 3)

$$P(a) + P(a)$$

 $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{213}$

e. P(5 or 6)

$$p(5) + P(4) = \frac{8}{52} = \frac{2}{13}$$

f. P(diamond or spade)

$$P(\lozenge) + P(\varOmega)$$

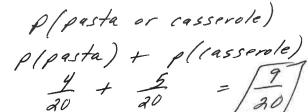
$$\frac{13}{52} + \frac{13}{52} = \boxed{\frac{1}{3}}$$

6. In a math class, 12 out of 15 girls are 14 years old and 14 out of 17 boys are 14 years old. What is the probability of selecting a girl or a 14-year old from this class?

15 girls
$$\rho(G) + \rho(14) - \rho(14) - p(14) = \frac{13}{33} = \frac{39}{33}$$
17 boys
$$\frac{15}{33} + \frac{36}{33} - \frac{13}{33} = \frac{39}{33}$$

7. The forecast predicts a 40% chance of snow on Tuesday and a 60% chance of snow on Wednesday. If these probabilities are independent, what is the chance it will rain on both days?

- 8. Travis places his favorite recipes in a bag for 4 pasta dishes, 5 casseroles, 3 types of chili, and 8 desserts.
 - a. If Travis choses one recipe at random, what is the probability that he selects a pasta dish or a casserole?



b. If Travis chooses one recipe at random, what is the probability that he does *not* select a dessert?

recipes

c. If Travis chooses two recipes at random <u>without replacement</u>, what is the probability that the first recipe he selects is a casserole and the second recipe he selects is a dessert?

$$\frac{5}{30} \cdot \frac{8}{19} = \frac{40}{380} = \left| \frac{2}{19} \right|$$

9. Morgan performs a magic trick in which she holds a standard deck of cards and has each of three people randomly choose a card from the deck. Each person keeps his or her card as the next person draws. What is the probability that all three people will draw a heart?

$$\frac{13}{53} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1716}{132600} = \frac{11}{850}$$



Tiles numbered 1 through 20 are placed in a box. Tiles numbered 11 through 30 are placed in a second box. The first tile is randomly drawn from the first box. The second tile is randomly drawn from the second box. Find each probability.



10.

P(both are greater than 15) $\frac{5}{30} \cdot \frac{15}{20} = \frac{75}{400} = \frac{3}{16}$



The first tile is odd and the second tile is less than 25. b. $\frac{10}{10} \cdot \frac{19}{10} = \frac{140}{200} = \left| \frac{7}{10} \right|$

The first tile is a multiple of 6 and the second tile is a multiple of 4.

The first tile is a multiple of 6 and the second tile is a multiple of 6, 13, 18,
$$\frac{3}{30} \cdot \frac{5}{30} = \frac{15}{400} = \frac{3}{80}$$

The first tile is less than 15 and the second tile is even or greater than 25.

$$\frac{14}{30} \cdot \frac{12}{35} = \frac{168}{500} = \frac{42}{135}$$

$$\frac{5econd \ tile}{9(even) + 9(>25) - 9(even + >25)}$$

$$\frac{10}{30} + \frac{5}{35} = \frac{12}{35}$$