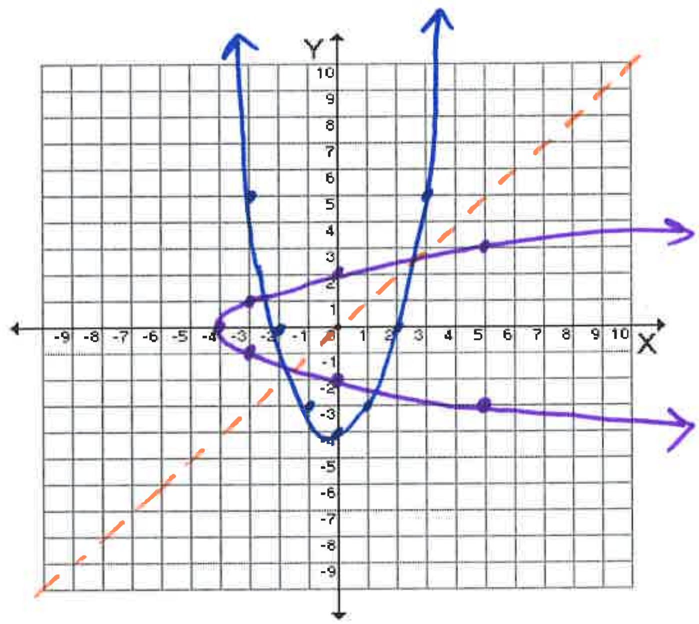
- Inverses relations interchange the x and y values
- The domain of the inverse is the range of the original function
- The range of the inverse is the domain of the original function
- Only one-to-one functions have inverses
- The graph of a function and its inverse are reflections of each other over the line y = x (mirror line)

Relation $y=x^2-4$

-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5
4	12

Inverse Relation

5	-3
0	-2
-3	-1
-4	0
-3	1
0	2
5	3
12	4



$y=x \rightarrow$ mirror line

*Not all functions have inverse functions

Horizontal Line Test – A function, f, has an inverse function, f^{-1} if and only if the graph passes the HLT.

Look at the following functions on your graphing calculator and determine whether its inverse function exists

a) $f(x)=|x-1|$ b) $f(x)=x^3-6x^2+12x-8$ c) $h(x)=\frac{4}{x}$

Find Inverse Functions Algebraically:

- Determine if it has an inverse
- replace f(x) with y
- Interchange x + y.
- Solve for y
- Replace y with $f^{-1}(x)$

$$f(x)=\frac{x-1}{x+2}$$

$$y=\frac{x-1}{x+2}$$

$$(y+2)x=\frac{y-1}{\cancel{y+2}}\cancel{(y+2)}$$

$$xy+2x=y-1$$

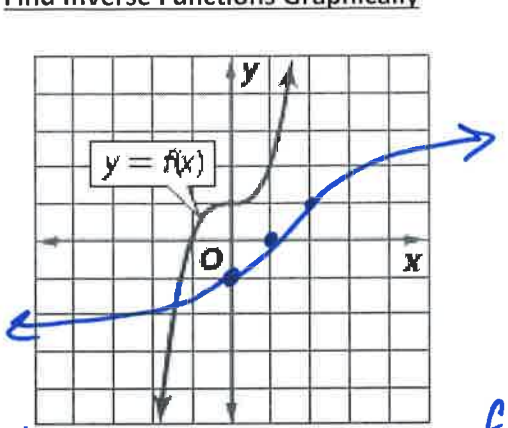
$$xy-y=-2x-1$$

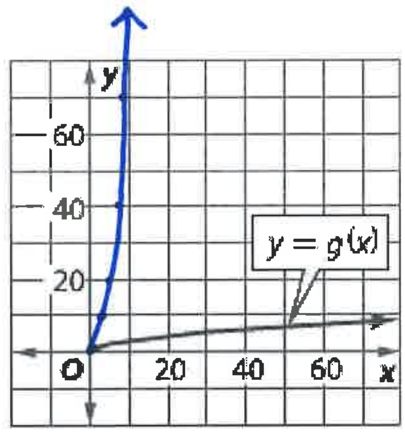
$$y(x-1)=-2x-1$$

$$y=\frac{-2x-1}{x-1}$$

$$\boxed{f^{-1}(x)=\frac{-2x-1}{x-1}}$$

Find Inverse Functions Graphically





$f(x)$

x	y
-1	0
01	0
1	2

$f^{-1}(x)$

x	y
0	-1
0	10
2	1

Verify Inverse Functions

Two functions, f and g, are inverse functions if and only if

- $[f \circ g](x) = x$
- $[g \circ f](x) = x$

$\boxed{Ex}$ Show that $f(x) = \frac{6}{x-4}$ and $g(x) = \frac{6}{x} + 4$ are inverses.

$$\overbrace{\frac{6}{\frac{6}{x}+4-4}}^{f(g(x))} = \frac{6}{\frac{6}{x}} = 6 \cdot \frac{x}{6} = \boxed{x}$$

The functions are inverses

$$\overbrace{\frac{6}{\frac{6}{x-4}}+4}^{g(f(x))} = \frac{\cancel{6}}{1} \cdot \frac{x-4}{\cancel{6}} + 4 = x-4+4 = \boxed{x}$$