

1-6 Function operations and composition of functions

Operations with functions
(add, subtract, multiply, or divide)

$$\text{Sum: } (f+g)(x) = f(x) + g(x)$$

$$\text{Difference: } (f-g)(x) = f(x) - g(x)$$

$$\text{Product: } (f \cdot g)(x) = f(x) \cdot g(x)$$

$$\text{Quotient } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} ; g(x) \neq 0$$

Examples: Given $f(x) = x^2 + 4x$, $g(x) = \sqrt{x+2}$, $h(x) = 3x - 5$
find each function and its domain.

a) $(f+g)(x)$

$$x^2 + 4x + \sqrt{x+2}$$

$$D: [-2, \infty)$$

b) $(f-h)(x)$

$$x^2 + 4x - (3x - 5)$$

$$x^2 + 4x - 3x + 5$$

$$x^2 + x + 5$$

$$D: (-\infty, \infty)$$

c) $(f \cdot h)(x)$

$$(x^2 + 4x)(3x - 5)$$

$$3x^3 - 5x^2 + 12x^2 - 20$$

$$3x^3 + 7x^2 - 20x$$

$$D: (-\infty, \infty)$$

d) $\left(\frac{h}{f}\right)(x)$

$$\frac{3x - 5}{x^2 + 4x}$$

$$D: x \neq 0, -4$$

$$(-\infty, 4) \cup (-4, 0) \cup (0, \infty)$$

Try this: $f(x) = x^2 + 6x - 8$ $g(x) = \sqrt{x}$

find each function and its domain

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f \cdot g)(x)$ d) $\left(\frac{f}{g}\right)(x)$

$$x^2 + 6x - 8 + \sqrt{x}$$
$$D: [0, \infty)$$

$$x^2 + 6x - 8 - \sqrt{x}$$
$$D: [0, \infty)$$

$$x^2\sqrt{x} - 6x\sqrt{x} - 8\sqrt{x}$$
$$D: [0, \infty)$$

$$\frac{x^2 + 6x - 8}{\sqrt{x}}$$
$$D: (0, \infty)$$

Compose Two Functions

one function is being used to evaluate a second function.

$[f \circ g](x) = f[g(x)] \rightarrow$ plug the g function in for all the x 's in the f function

$[g \circ f](x) = g[f(x)] \rightarrow$ plug the f function in for all the x 's in the g function.

Ex) given $f(x) = x^2 + 1$ and $g(x) = x - 4$ find...

a) $[f \circ g](x) = f(g(x))$

b) $[g \circ f](x) = g(f(x))$

$$(x-4)^2 + 1$$
$$(x-4)(x-4) + 1$$

$$(x^2 + 1) - 4$$
$$x^2 - 3$$

$$x^2 - 8x + 17$$

Combinations

Evaluate at a number

- do the indicated operation, then substitute

Ex: $f(x) = x^2 - 9$ $g(x) = 4x + 7$

find $(f \circ g)(-2)$

$$(x^2 - 9)(4x + 7)$$

$$4x^3 + 7x^2 - 36x - 63$$

$$4(-2)^3 + 7(-2) - 36(-2) - 63$$

$$-32 - 14 + 72 - 63$$

$$\boxed{-37}$$

Compositions

Evaluate at a number

Ex $f(x) = x^2 + 2x$ $g(x) = 9x$

find $[f \circ g](7) = f(g(7))$ * work from inside out

$$\begin{array}{l} \underbrace{g(7)} \\ 9(7) \\ 63 \end{array} \rightarrow \begin{array}{l} \underbrace{f(63)} \\ (63)^2 + 2(63) \\ 3969 + 126 \end{array}$$

$$\boxed{4095}$$